CHANGE-OVER DESIGNS

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Abstract: The experiments in which each experimental unit receives some or all of the treatments, one at a time, over a number of periods are known as change-over designs (CODs). These designs are also called cross-over designs, switch-over trials, time-series designs, before-after designs, repeated measurements designs, designs involving sequences of treatments, *etc*. These designs have been used advantageously in several fields of research, notably in nutrition experiments with dairy cattle, clinical trials in medical research, psychological experiments, long-term agricultural field experiments and bioequivalence trials. Some aspects of CODs are discussed here. Some methods of constructing these designs considering direct and first order residual effects have been described. Besides this, some methods of construction of incomplete sequence balanced CODs for direct and first order residual effects are also presented.

Key words: *Change-Over Design, Direct Effect, Residual Effect, Circular Change-Over Design, Variance Balanced, Totally Balanced, Incomplete Sequence Balanced Change-Over Design*.

1. Introduction

Designing an experiment implies deciding how the experimental units are grouped, the treatments are allocated to these and the observations or measurements are taken to solve a particular problem in a valid, efficient and economic way. In experiments where the experimental units are human beings or animals, error variation cannot be controlled by grouping / blocking as it also involves the physiological and genetic dissimilarities of the units. Hence to eliminate such variation, it is desirable to design experiments in such a manner that each experimental unit receives some or all of the treatments, one at a time, over a certain period of time. Such designs are known as changeover designs (CODs) or repeated measurements designs or crossover trials or switchover trials or designs involving sequences of treatments or time-series designs or before-after designs. These designs have been used advantageously in several fields of research, notably in nutrition experiments with dairy cattle, clinical trials in medical research, psychological experiments, long-term agricultural field experiments and bioequivalence trials. A good review of different classes of these designs can be seen in Afserinejad (1990) and Hinkelmann and Kempthorne (2005).

Cochran *et al*. (1941) suggested a procedure for comparing three treatments (rations) 1, 2 and 3 which were fed to the cows (experimental units) in sequences determined by the columns of the squares. Arrangement of the cows in blocks defined by squares usually leads to an increase in accuracy. The treatments are randomly allotted to the rations and in each block the sequences are randomly assigned to the cows.

Periods	Experimental units					
	I	II	III	IV	V	VI
1	1	2	3	1	2	3
2	2	3	1	3	1	2
3	3	1	2	2	3	1

Need for Change-Over Designs: There are several situations listed as follows, where it is essential to go for CODs:

(i) Experiments that involve biological entities, such as animals or human beings where such entities exhibit larger variability.

(ii) Due to budget constraint, the experimenter has to use each experimental unit on several occasions.

(iii) In some experiments, the treatment effects do not have a serious damaging effect on the experimental units and hence these units can be used for successive occasions.

(iv) In some experiments, the experimental units are human beings or animals and often the nature of the experiments is such that it calls for special training over a long period of time. Therefore, due to time limitations one is forced to use these experimental units for several tests.

(v) When one of the objectives of the experiments is to find out the effect of different experimental units as in drug, nutrition or learning experiments.

(vi) Sometimes the experimental units are scarce; therefore, experimental units have to be used repeatedly.

Residual Effects: The peculiarity of a COD is that any treatment applied to a unit in a certain period influences the responses of the unit not only in the period of its application but also leaves residual effects in the following periods. These residual effects or carry-over effects may be of different magnitudes. Residuals which persist only for one succeeding period, are called first order residual effects or simply, first residual effects. In general, the k^{th} order residual effect is one, which persists up to k successive periods.

Direct Effect: The effect of a treatment in the period of its application (devoid of residual effect, if any) is known as the direct effect of the treatment.

The simplest type of COD is a Latin square design or a set of squares with rows representing periods of time and columns the experimental units. Provided that there are no carry-over or residual effects and experimental conditions are not such that treatment effects vary to any marked extent from one period to another, the ordinary analysis of Latin squares can be used. But it is always desirable to allow for the possibility that residual effects exist. An allowance for residual effects can be made in the design and by introducing additional constants for residual effects in the model; the analysis of the data can be carried out.

2. Designs with First Order Residual Effects

2.1 Model

Let a COD be represented as COD (v, p, n) where v treatments are arranged in p periods and n experimental units. Considering the first residual effects of treatments, the additive fixed effects model can be written as

$$Y_{hijk} = \mu + \pi_h + \tau_i + \rho_j + \psi_k + \varepsilon_{hijk,} \quad ...(2.1)$$

$$h = 1,..., p;\ i, j = 1,..., v;\ k = 1,..., n;$$

where Y_{hijk} is the observation from the k^{th} experimental unit in the h^{th} period when treatment i is applied to it and is preceded by the j^{th} treatment in $(h-1)^{th}$ period (h>1) and μ , π_h , ψ_k , τ_i , and ρ_j represent the general mean, effect of the h^{th} period, effect of the k^{th} unit, direct effect of treatment i and residual effect of the treatment j, respectively, ε_{hijk} are random errors assumed to be identically and independently distributed with $N(0, \sigma^2)$. ρ_j = 0 for all values of j for the observations in the first period.

The model may be expressed in matrix notation as:

$$\mathbf{Y} = \mu\,\mathbf{1} + \mathbf{P}\,\boldsymbol{\pi} + \mathbf{T}\boldsymbol{\tau} + \mathbf{R}\,\boldsymbol{\rho} + \mathbf{S}\,\boldsymbol{\psi} + \boldsymbol{\varepsilon} \quad ...(2.2)$$

where,
$\mathbf{Y}$ = np × 1 vector of observed responses,
$\boldsymbol{\pi}$ = p × 1 vector of period effects,
$\boldsymbol{\tau}$ = v × 1 vector of direct effects of treatments,
$\boldsymbol{\rho}$ = v ×1 vector of residual effects,
$\boldsymbol{\psi}$ = n × 1 vector of unit effects,
$\mathbf{1}$ = np × 1 vector of unities,
$\mathbf{P}$ = np × p design matrix of observations vs. periods,
$\mathbf{T}$ = np × v design matrix of observations vs. direct effects of treatments,
$\mathbf{R}$ = np × v design matrix of observations vs. residual effects of treatments,
$\mathbf{S}$ = np × n design matrix of observations vs. experimental units,
$\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_{np})$ is np × 1 vector of random errors that follows $\mathbf{N}(0,\sigma^2)$

2.2 Uniform CODs

A COD is called uniform on periods if each treatment occurs in each period the same number of times, say λ_1. A necessary condition for this to hold is that $n = \lambda_1 v$. A design is called uniform on units, if each treatment is applied to each experimental unit the same number of times, say λ_2. This can occur only if $p = \lambda_2 v$. A design is called uniform if it is uniform on both periods and units. A Williams square COD can be considered as a uniform COD. For v = 5 treatments a Williams square COD is as follows:

	First Latin square					Second Latin square				
Periods	Experimental units					Experimental units				
	I	II	III	IV	V	I	II	III	IV	V
1	1	2	3	4	5	5	1	2	3	4
2	5	1	2	3	4	1	2	3	4	5
3	2	3	4	5	1	4	5	1	2	3
4	4	5	1	2	3	2	3	4	5	1
5	3	4	5	1	2	3	4	5	1	2

2.3 Balanced CODs

The analysis is made easier if each treatment is preceded equally often by every other treatment. Designs having this property have been called *combinatorially balanced* with respect to first residual effects. A COD permitting the estimation of first order residual effects, is called *variance balanced* if the variance of any elementary contrast among the direct effects is constant, say α, and the variance of any estimated elementary contrast among the residual effects is also constant, say β. Here, α and β may not be equal. If $\alpha = \beta$, then these designs are known as *totally balanced* CODs. It may be noted here that a combinatorially balanced COD may not be variance balanced.

2.4 Extra-Period CODs

The balanced change-over designs may not be desirable always because:

(i) They do not estimate direct and residual effects of treatments independently, for the estimates are correlated.

(ii) They give less precise estimates of residual effects than of direct effects.

Designs that give independent estimates of direct and residual effects, of approximately equal precision, are obtained by adding an extra period to the original design. In the new extra period, the treatments that were applied in the previous final period are repeated. These designs are called extra-period balanced CODs. The following is an extra-period balanced COD for 3 treatments 1, 2 and 3 on 6 units in 4 periods:

	Experimental Units					
Periods	I	II	III	IV	V	VI
1	1	2	3	1	2	3
2	2	3	1	3	1	2
3	3	1	2	2	3	1
4	3	1	2	2	3	1

Any treatment is now preceded equally often by every treatment including itself. This makes estimation of the contrasts of direct and residual effects orthogonal.

2.5 Pre-Period CODs

Pre-period or 0^{th} period is a period preceding the first experimental period in which appropriate treatments are applied to the experimental unit but either the observations are not recorded or if recorded they are not taken into consideration while carrying out the analysis of the data. This pre-period with appropriate treatment makes the observation homogenous in the sense that the first period observations also contain the residual effects of the treatments thereby increasing the orthogonality in the data. The following is a pre-period design balanced for 4 treatments 1, 2, 3 and 4:

	Experimental Units			
Periods	I	II	III	IV
0	1	2	3	4
1	1	2	3	4
2	4	1	2	3
3	2	3	4	1
4	3	4	1	2

2.6 Circular CODs for Estimation of Direct and First Order Residual Effects

A COD allowing the estimation of direct and first order residual effect with one pre-period (observations are not recorded during pre-period), in which the treatments in the pre-period are exactly the same as those in the last period in the sequence, will be called a circular COD for the estimation of direct and first order residual effects.

A circular COD with 7 treatments 14 units and 3 periods with one pre-period for the estimation of direct and first order residual effects is as follows:

		Experimental units													
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
Periods	0	4	5	6	7	1	2	3	5	6	7	1	2	3	4
	1	1	2	3	4	5	6	7	3	4	5	6	7	1	2
	2	2	3	4	5	6	7	1	6	7	1	2	3	4	5
	3	4	5	6	7	1	2	3	5	6	7	1	2	3	4

2.7 Method of Construction of Totally Balanced CODs

Varghese and Sharma (2000) have given a method of constructing a class of complete sequence totally balanced CODs considering the first order residual effects of treatments. A method of construction of a class of incomplete sequence totally balanced CODs using a special series of Balanced Incomplete Block designs has been given by Dey and Balachandran (1976).

2.7.1 Method of Construction of Complete Sequence Totally Balanced CODs

For constructing a complete sequence totally balanced COD for v treatments in v units and (2v-1) periods, the procedure is as follows:

Denote the v treatments under test by 0, 1, 2…, v-1. A sequence of 2v elements is formed by interlacing the elements of the sequence {0, 1, 2, 3…v-1} with the elements of its reverse {v-1,v-2,…, 2, 1, 0},viz.

{0, v-1, 1, v-2, 2, v-3,…, v-2, 1, v-1, 0}.

Delete one of the two middle terms having the same value either v/2 or (v-1)/2 according as v is even or odd. The sequence thus formed is called the initial sequence. On developing this sequence mod (v), we get an array of (2v-1) rows and v columns. If the columns of this array represent the experimental units and rows the periods, we get a design with (2v-1) periods and v experimental units. However, this design is not variance balanced. In order to make this design variance balanced, a pre-period is added with treatments exactly the same as those in the last period. Thus the required COD (v, 2v-1, v) is a pre-period design with (2v-1) periods and v experimental units for v treatments. An interesting feature of this design is that it is symmetrical about the middle v^{th} period.

Example 2.7.1.1.: Let v = 4, then p = 2v-1 =7 and n = v = 4. The initial sequence is {0, 3, 1, 2, 1, 3, 0}. On developing this sequence mod (4) and adding a pre-period with treatments (0, 1, 2, 3), the following totally balanced COD (4, 7, 4) is obtained (the period numbered 0 is the pre-period):

Periods	

Experimental units

	I	II	III	IV
0	0	1	2	3
1	0	1	2	3
2	3	0	1	2
3	1	2	3	0
4	2	3	0	1
5	1	2	3	0
6	3	0	1	2
7	0	1	2	3

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2.7.2 Method of Construction of Incomplete Sequence Totally Balanced CODs

We denote the parameters of a BIB design by the usual symbols v, b, r, k, λ. Sprott (1954) has shown that if v (= mk+1) is a prime or a prime-power, then the following series of BIB designs can be constructed:

$$v = mk+1,\ b = mv,\ r = mk,\ k,\ \lambda = k-1 \qquad \ldots(2.7.2.1)$$

The design (2.7.2.1) can be constructed by developing mod v, each of the following m initial blocks:

$$(x^i, x^{i+m}, x^{i+2m}, \ldots, x^{i+(k-1)m}),\ i = 0, 1, \ldots, m-1 \qquad \ldots(2.7.2.2)$$

x being a primitive element of GF (v)

The series (2.7.1.1) of BIB designs is used to construct totally balanced CODs. If the blocks of the above BIB design are written as columns, we get a COD in v treatments, n = mv sequences and p = k periods, where columns are treated as sequences and rows as periods. This COD is not balanced. For making it balanced the last treatment in each sequence is placed in the pre-period thus making the design circular. Thus, from the BIB design (2.7.1.1) an arrangement of k+1 rows (periods) and mv columns (sequences) are obtained where the columns of the arrangement are the blocks of the BIB design (2.7.2.1). The series of designs thus obtained are totally balanced.

Example 2.7.2.1: Let m =2, k =3, so that v = 7. The COD in 14 sequences and 3 periods is as follows:

		Experimental Units													
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
Periods	0	4	5	6	7	1	2	3	5	6	7	1	2	3	4
	1	1	2	3	4	5	6	7	3	4	5	6	7	1	2
	2	2	3	4	5	6	7	1	6	7	1	2	3	4	5
	3	4	5	6	7	1	2	3	5	6	7	1	2	3	4

2.8 Method of Construction of Balanced CODs: An easy method of constructing Latin square CODs which are balanced for the direct and first order residual effects is given by Sharma (1975). These designs are complete sequence CODs. When number of available periods in the experiment is not able to accommodate all the treatments then we go for Incomplete Sequence CODs and construct CODs which are balanced for the estimation of

direct and residual effects of treatments. A class of such designs is given by Patterson and Lucas (1962).

2.8.1 Method of Construction of Complete Sequence Balanced CODs

The different steps involved in the construction of complete sequence balanced CODs are as follows:

(i) Construct two v×v tables in which columns refer to individuals and rows refer to the order of presentation.
(ii) In both the squares, number the order of presentation from 1 to v successively.
(iii) Number the treatments i = 0, 1, 2,..., v-1.
(iv) Assign these treatments successively to the v cells in the first column of both the squares by proceeding from top to bottom, entering only in odd-numbered cells in the first and even numbered cells in the second square, and then reversing the direction, filling in even-numbered cells in the first and odd-numbered cells in the second square.
(v) Obtain the successive columns of the squares by adding integer 1 to each element of the previous column and reducing the elements, if necessary, by mod v.

It is to be noted that in each of the constructed squares every treatment occurs in each row and in each column precisely once. Moreover, when v is even, each treatment is preceded exactly once by other treatment in either of the two squares. Thus, in this case either of the two squares may be used. This situation occurs in neither of the two squares if v is odd. However, when both the squares are considered together, each treatment is preceded by every other exactly twice. Consequently, both the squares must be used in this case.

Example 2.8.2.1: Latin squares for v = 4 are given below:

First square					Second square				
Order of presentation	Individual				Order of presentation	individual			
	I	II	III	IV		I	II	III	IV
1	0	1	2	3	1	3	0	1	2
2	3	0	1	2	2	0	1	2	3
3	1	2	3	0	3	2	3	0	1
4	2	3	0	1	4	1	2	3	0

Example 2.8.2.2: Latin squares for v = 5 are obtained as follows:

First square						Second square					
Order of presentation	Individual					Order of presentation	Individual				
	I	II	III	IV	V		I	II	III	IV	V
1	0	1	2	3	4	1	4	0	1	2	3
2	4	0	1	2	3	2	0	1	2	3	4
3	1	2	3	4	0	3	3	4	0	1	2
4	3	4	0	1	2	4	1	2	3	4	0
5	2	3	4	0	1	5	2	3	4	0	1

2.8.2 Method of Construction of Incomplete Sequence Balanced CODs

Patterson and Lucas (1962) developed Incomplete Sequence Balanced COD using the block contents of each of the block of the BIBD. Take the first block and construct Williams Squares for the treatments in that block. So we get one or two Williams Squares from one block depending upon whether the block size is even or odd. Likewise, we get as many numbers of Williams Squares as the number of blocks, if the block size is even. The number of Williams squares will be doubled the number of blocks, if the block size is odd. These all blocks constitute a complete set of incomplete sequence balanced COD.

Example 2.8.2.1: For 5 treatments and block size 4, we consider the following BIBD with parameter (5, 5, 4, 4, 3):

1 2 3 4
1 2 3 5
1 2 4 5
1 3 4 5
2 3 4 5

The incomplete sequence balanced COD formed by using the block contents of this BIB design is:

		Experimental units											
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Periods	1	1	2	3	4	1	2	3	5	1	2	4	5
	2	4	1	2	3	5	5	2	3	5	1	2	4
	3	2	3	4	1	2	3	5	1	2	4	5	1
	4	3	4	1	2	3	4	1	2	4	5	1	2

		Experimental units							
		XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Periods	1	1	3	4	5	2	3	4	5
	2	5	1	2	4	5	2	3	4
	3	3	4	5	1	3	4	5	2
	4	4	5	1	2	4	5	2	3

Thus we get incomplete sequence balanced COD with v = 5, p = 4 and n = 20.

2.9 Minimal Balanced CODs

A balanced COD (v, p, n) is said to be minimal if its parameter λ_1 (number of times each treatment occurs equally frequently) is the smallest integer such that $\lambda_1 (p-1) = 0$ [mod (v-1)]. Thus, a minimal balanced design contains the minimum possible number of experimental units. A general method of construction of these designs for all odd v has given by Sharma *et al.* (2003).

2.9.1 Method of Construction

Let v = (2t+1) treatments be arranged in their natural order 0, 1, 2, 3, ..., 2t.

Case 1 (t odd). Obtain two sequences of treatments, first by interlacing the last (t+1)/2 treatments in their reverse order within the first (t+1)/2 treatments, and the second by

interlacing the first (t+1)/2 treatments within the reverse order of last (t+1)/2 treatments. On developing these sequences mod v, an arrangement of v treatments in 2v columns and (v+1)/2 rows is obtained. This arrangement forms a minimal balanced COD [v, (v+1)/2, 2v] design with columns representing experimental units and rows the periods.

Example 2.9.1: Suppose v =11, t =5. We retain the first (t+1)/2 =3 treatments (0, 1, 2) and the last (t+1)/2=3 treatments (8, 9, 10).

First sequence: {0, 10, 1, 9, 2, 8} Second sequence: {10, 0, 9, 1, 8, 2}

Developing these sequences mod 11, the following design is obtained:

Periods	Experimental units										
	1	2	3	4	5	6	7	8	9	10	11
1	0	1	2	3	4	5	6	7	8	9	10
2	10	0	1	2	3	4	5	6	7	8	9
3	1	2	3	4	5	6	7	8	9	10	0
4	9	10	0	1	2	3	4	5	6	7	8
5	2	3	4	5	6	7	8	9	10	0	1
6	8	9	10	0	1	2	3	4	5	6	7

Periods	Experimental units										
	12	13	14	15	16	17	18	19	20	21	22
1	10	0	1	2	3	4	5	6	7	8	9
2	0	1	2	3	4	5	6	7	8	9	10
3	9	10	0	1	2	3	4	5	6	7	8
4	1	2	3	4	5	6	7	8	9	10	0
5	8	9	10	0	1	2	3	4	5	6	7
6	2	3	4	5	6	7	8	9	10	0	1

Case 2 (t even). The first sequence is obtained by interlacing the last t/2 treatments in their descending order within the first (t/2)+1 treatments in their natural order. The second sequence is obtained by interlacing the first t/2 treatments in their natural order within the last (t/2) +1 treatments in their descending order. On developing these sequences mod v, a minimal balanced COD [v, (v+1)/2, 2v] design is obtained with columns representing experimental units and rows the periods.

Example 2.9.2: Let v = 9 giving t = 4. Arrange these treatments as 0, 1, 2,…, 8. The first sequence {0, 8, 1, 7, 2} is obtained by interlacing 8, 7 within 0, 1, 2 and the second sequence {8, 0, 7, 1, 6} is the result of interlacing 0, 1 within 8, 7, 6. These sequences when developed mod 9 yield the minimal balanced COD (9, 5, 18) design as follows:

Periods	Experimental units								
	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	8	0	1	2	3	4	5	6	7
3	1	2	3	4	5	6	7	8	0
4	7	8	0	1	2	3	4	5	6
5	2	3	4	5	6	7	8	0	1

	Experimental units

Periods	10	11	12	13	14	15	16	17	18
1	8	0	1	2	3	4	5	6	7
2	0	1	2	3	4	5	6	7	8
3	7	8	0	1	2	3	4	5	6
4	1	2	3	4	5	6	7	8	0
5	6	7	8	0	1	2	3	4	5

These designs are not variance balanced. In fact, these designs are partially balanced CODs.

2.10 Strongly Balanced CODs

Designs in which each treatment is preceded by every other treatment as well as by itself equally often are called strongly balanced CODs. These designs estimate direct effects of the treatments orthogonal to all the other effects.

2.10.1 Method of Construction

Quenouille (1953) has given a strongly balanced design for two treatments. It can be formed by taking all possible cyclical arrangements (within subjects) of the treatments1, 1, 2, 2. Berenblut (1964) has given a method of constructing design for v treatments. The method is as follows:

Let
α = A B C D…U V
β = V A B C…T U
γ = U V A B…S T
……
……
ψ = D E F G…B C
ϕ = C D E F…A B
ω = B C D E…V A

If v is odd, the design for v treatments can be written symbolically as:

	Subject (1 to v^2)	
1	α	α… α
2	β	γ... α
3	γ	γ… γ
4	δ	ε… γ
5	ε	ε… ε
.	.	…..
.	.	…..
Period v-1	ϕ	ω…ψ
v	ω	ω… ω
v+1	ω	α… ϕ
v+2	ϕ	ϕ… ϕ
.	.	……
.	.	……
2v-1	β	β… β
2v	α	β… ω

If v is even, the lines for periods v and v+1, for periods v-1 and v+2, etc., are interchanged. For all values of v, the number of subjects required is v^2 and the number of periods 2v.

The following variations of the design are possible without upsetting the balance:
(i) Rows with odd numbers may be permuted amongst themselves (v! permutations).
(ii) Rows with even numbers may be permuted amongst themselves (v! permutations).
(iii) The design may be read in reverse order in time.

Example 2.10.1: For 3 treatments in which successive rows can be written in the form ααα, γβα, βββ, βαγ, γγγ, αγβ where α =1 2 3, β = 2 3 1, γ = 3 1 2, the design obtained is as follows:

Period	Subject								
	I	II	III	IV	V	VI	VII	VIII	IX
1	1	2	3	1	2	3	1	2	3
2	3	1	2	2	3	1	1	2	3
3	2	3	1	2	3	1	2	3	1
4	2	3	1	1	2	3	3	1	2
5	3	1	2	3	1	2	3	1	2
6	1	2	3	3	1	2	2	3	1

Subsequently Sharma (1981) constructed a class of nearly strongly balanced CODs. These designs required only v experimental units for 2v periods for v treatments. In these designs the treatment effects (direct and residual effects) are estimated with nearly the same precision as in Quenouille (1953), Barenblut (1964) and Patterson (1973) designs.

3. COD with Two Treatments

Experiment in which two treatments are applied over periods to the same experimental units is called COD with two treatments. COD with two treatments have received special attention of the residual effects. Grizzle (1965), Kershner and Federer (1981) and Mathews (1987) studied these designs with small number of periods. Subsequently, Kunert (1991) derived two-treatment CODs for experiments with many periods. Some important two treatment CODs with a suitable pre-period are given by Varghese *et al*. (2000), schematically presented in the following table. The schematics give the placement of the treatments with respect to the periods (rows) and sequences (columns) of the designs.

Design No.	(v, p, n)	Design				
D_{01}	(2, 2, 4)	0	A	B	B	A
		1	A	B	A	B
		2	B	A	A	B

D_{02}	(2, 3,4)	0	B	A	A	B
		1	A	B	A	B
		2	A	B	B	A
		3	B	A	B	A

D_{03}	(2, 3, 6)	0	B	A	A	B	A	B

		1	A	B	A	B	A	B
		2	A	B	B	A	B	A
		3	B	A	B	A	A	B

6. Conclusions

Designs in which each experimental unit receives a sequence of several treatments in successive periods are known as change-over designs. These designs can be advantageously used where the experimental unit are highly variable and the treatments exhibit residual effects. These designs allow the estimation of treatment effects (direct treatment effects and residual treatment effects) contrasts free of unit effects. When number of available periods in the experiment is not able to accommodate all the treatments then we use incomplete sequence balanced CODs which may be balanced or partially balanced.

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